Retirement Planning Project Name: _____

**Part I. TVM Formulas**

The time-value-of-money formula for a future value ($FV$) as a function of a present value ($PV$), interest rate ($r$), and number of periods of compounding interest ($n$).

$$FV = PV(1+r)^n$$

If you consider depositing the same amount ($PMT$) each year, your future value formula is more complex. Assuming you do not earn interest the year you make the deposit, you will have $n$ payments, each earning interest for the remaining years so:

$$F = PMT(1+r)^{n-1} + PMT(1+r)^{n-2} + ... + PMT(1+r)^1 + PMT$$

a. Rearrange the terms above to derive a formula for a future value of a series of $n$ equal annual payments ($PMT$) that earn annual interest ($r$). You will need this identity:

$$\sum_{i=1}^{n}(1+r)^{i-1} = \frac{(1+r)^n - 1}{r}$$

This equation tells you how much money will be in your portfolio after $n$ years of saving $PMT$ each year and earning interest $r$ each year.

$FV =$ _____

b. Rearrange the terms in the first formula to derive an equation for the present value of a future value discounted at an interest rate ($r$) for $n$ periods.

$PV =$ _____

c. Combine the equations in parts (a) and (b) to derive a formula for the present value of a series of $n$ equal annual payments/deposits ($PMT$) that earn annual interest ($r$). This determines how much money you need in your portfolio in order to live off $PMT$ each year (while still earning $r$ interest on the portfolio).

$PV =$ _____

**Part II. What You Need**

a. Estimate your annual budget to live comfortably (by your definition). You can use any source you'd like to estimate expenses, including Table 5 (Average Annual Expenditures and Characteristics) from the Bureau of Labor Statistics "Consumer Expenditures Survey, 2014" (Report #1063, Oct 2016), pp20-22.
https://www.bls.gov/opub/reports/consumer-expenditures/2014/pdf/home.pdf

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(Add rows if necessary.)

Total Annual Expenses = _____

b. Estimate how long you will live on your savings in retirement. (For example, if you retire at $65$ years old and expect to live to $85$, you will need $85 - 65 = 20$ years' worth of savings.)

$n$ = _____

c. Assuming you're a wise investor and you earn an after-tax, real return of $2\%$ on your retirement portfolio, calculate how large your portfolio must be in order to draw your annual expenses from (IIa) for (IIb) years. Use the formula from (Ic)... you can check your formula using the tvm_PV app on your TI-83 or 84 calculator or the PV function in Excel.

$PV$ = _____

**Part III. What You Must Save**

a. Estimate how many years you plan to work between now and retirement. (For example, if you are $20$ years old now and plan to work to age $65$, you have $65 - 20 = 45$ years of working... and saving!).

$n$ = _____

b. The lowest real rate of return on the S&P 500 index over a $45$-year period is $4.4\%$. You'll likely earn much more in most years, but may also earn less during market contractions. Using $4.4\%$ as a "safe" estimate of earnings for your investments, determine how much you have to save each working year, for (IIIa) years, in order to build up your portfolio to the value you found in

(IIb). You will have to rearrange the terms in your answer to (Ia). You can check your formula using the tvm_Pmt app on your calculator or the PMT function in Excel. (NOTE: The PV you solved in (IIb) is now a FV as you work and save to retirement.)

$PMT =$ _____

**Part IV. Timing vs Time**

Suppose you have a $\$2{,}000$ balance that grows at $8\%$ interest, compounded annually.

a. Calculate the account balance after $10$ years.

$F =$ _____

b. Suppose someone offers you a "better" investment if you withdraw the money and transfer it into a different account. You will earn no interest this year, but in future years you will earn $8\%$ compounded monthly. What is the effective rate?

$r_e =$ _____

c. What is the account value at the end of $10$ years. (Note, you only have $9$ years of interest.) Is this a better investment?

$F =$ _____

Better? Yes or No

d. How long does this investment have to earn interest in order for the higher rate to drive the account balance above the balance of the lower rate account? You can solve this mathematically (logs help) or use trial and error in Excel. Round up to the nearest integer.

$n =$ _____

e. What is the lesson here? (See *Basic Personal Finance* p90).

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